Journal of Engineering Physics and Thermophysics, Vol. 79, No. 2, 2006

NUMERICAL INVESTIGATION OF THE LAMINAR SHOCK-WAVE GAS FLOW EXCITED BY HARMONIC VIBRATIONS OF THE PISTON IN A CLOSED CYLINDRICAL TUBE

S. V. Bulovich UDC 534.222

The shock-wave gas flow in a closed cylindrical tube caused by harmonic vibrations of a flat piston has been considered. The solution has been obtained by means of numerical integration of the system of nonstationary equations of a narrow channel written in divergent form. A scheme of the 1st order of time accuracy and a scheme of the 2nd order of space accuracy have been used. The regularities in the behavior of the dynamic and temperature boundary layers for the unstable regime of gas flow in the tube at various moments of the cycle have been analyzed.

The wave gas flow in a closed tube has been the subject of numerous investigations. A certain systematization of the information is given in monographs [1] and [2]; recent results are presented in [3]. The above-mentioned works contain reviews of not only the experimental studies but also various mathematical models that were used to describe the circle of phenomena in question. The range of their applicability is given.

It is stated that, at present, numerical simulation of a nonstationary gas flow within the framework of the complete system of Navier–Stokes equations is only possible for relatively short channels. Examples are the recent studies [4] and [5]. For channels having large elongations, adequate account of the transfer along the channel by means of the diffusion process within the framework of a numerical simulation seems to be problematic. Therefore, particular forms of asymptotic approximation are used. In particular, as the description becomes less complete compared to the simulation within the framework of the complete system of Navier–Stokes equations, the so-called parabolic approximation [6], where only the processes of longitudinal diffusion are neglected, is distinguished. The next model in the hierarchy is the parabolized approximation [7]. Here, besides neglecting the diffusion along the channel, the pressure field is represented as the sum of two functions, one of which — the one used to describe the pressure gradient in the axial direction — depends only on this coordinate. This technique permits excluding the mutual influence of the longitudinal and transverse pressure gradients. Despite the simplifications compared to the complete system of Navier–Stokes equations, the employment of these models to describe the nonstationary processes of gas oscillation in a tube has not yet been described in the literature.

Complete refusal to take into account the transverse pressure gradient leads to a system of a narrow channel equations [8]. Examples of the successful use of such a system of equations for oscillatory fluid motion in a tube are the investigations described in [9] and [10]. The conventional hierarchical list of mathematical models is closed by the quasi-one-dimensional approach to the description of the gas flow in the channel.

In turn, already at the stage of problem formulation, each of the above models admits reduction in taking into account the properties of the gas compressibility. Moreover, when the influence of the flow rate on the gas state is completely taken into account, one distinguishes between the hyposonic approximation and the incompressible liquid approximation.

The applicability limits of a particular approximation are primarily determined by the channel geometry and the flow conditions, while for the parabolic and parabolized Navier–Stokes equations a restriction is imposed only on the value of the Reynolds number, and for the system of narrow channel equations an additional requirement is the smoothness of the change in the functions on the channel wall. This requirement is applied to the value of the local

---

St. Petersburg State Technical University, 29 Politekhnicheskaya Str., St. Petersburg, 195251, Russia; email: bulovich@ista-pneumatics.ru. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 79, No. 2, pp. 139–144, March–April, 2006. Original article submitted October 29, 2004; revision submitted March 12, 2005.

 1062-0125/06/7902-0354

slope of the wall element with respect to the direction of the channel axis, the distribution of the mass transfer through the tube surface, and the dependence of the thermal state of the tube wall along the channel. For the quasi-one-dimensional approximation model, apart from the above limitations, a priori information on the dependence of velocity and temperature profiles in the channel on the flow conditions is needed. Such dependences have been established for steady-state regimes of gas flow. Therefore, simulation by the quasi-one-dimensional model finds no application beyond the scope of the quasi-stationary flow. In the present paper, we concentrate on the narrow channel approximation.

Consider the problem on the wave gas flow, for definiteness — air flow, in a cylindrical tube of diameter D and length L. One end of the tube (the left-hand one) is plugged and on the other (right-hand) end there is a flat piston executing oscillations by the harmonic law with amplitude h and oscillation frequency ω.

For the problem formulation under consideration, we can distinguish three dynamic similarity numbers — the Reynolds number Re, Stokes number St, and Mach number M — and two thermal similarity numbers — the Prandtl number Pr and the temperature factor Θ, which define the whole diversity of regimes of gas flow in the tube. Let us express the similarity numbers in terms of the problem parameters as follows:

$$\mathrm{Re} = \frac{h\omega D}{\nu}, \quad \mathrm{St} = D\sqrt{\frac{\omega}{\nu}}, \quad \mathrm{M} = \frac{h\omega}{c}, \quad \mathrm{Pr} = \frac{\nu}{a}, \quad \Theta = \frac{T_0}{T_\mathrm{w}}.$$

Three important points that take place when the gas is moving under the action of piston oscillations are noteworthy. First, at a circular frequency multiple of the quantity $\pi c L^{-1}$ in the system there occurs excitation of free oscillations of the gas column and the phenomenon of resonance independent of the piston oscillation amplitude is observed. Second, beginning with a certain level of the oscillation amplitude and simultaneously at a near-resonance frequency range, the sinusoidal behavior of the functions along the channel is replaced by discontinuity solutions. The transition to discontinuity solutions is described only in experimental works. The region of reverse transition from the shock waves to the shock-free wave gas flow has been established experimentally only for situations associated with the deviation of the oscillation frequency from the resonance frequency. The threshold oscillation amplitude, when the shock wave still exists, has not been investigated experimentally. Theoretical works on this topic are completely absent. Third, it has been established that for flows without discontinuities at Reynolds numbers exceeding 700St the laminar regime of gas flow is replaced by the turbulent one. Note that in the case of reciprocating motion of the medium the critical Reynolds number considerably exceeds its value for the same tube under stationary motion. The transition from the laminar form of motion to the turbulent one has been described in a number of experimental works. However, the mechanism of the transition in the form of theoretical dependences has not been established.

The gas oscillation in a tube has received the best study for $\mathrm{M} \ll 1$. In this case, the equations describing the gas behavior become linear. Any finite value of the Mach number leads to a nonlinear system of equations. Either direct linearization of this system or construction of the solution in the form of a series, where the Mach number is an expansion parameter, are possible. Here the calculation of the gas oscillations at the resonance frequency is of interest by virtue of the fact that a satisfactory solution cannot be obtained in terms of analytic representation. Linearization of the system of nonlinear equations leads to equations that have a solution at small oscillations when no shock wave propagating along the channel is formed. Examples of analytic solutions as applied to the small-amplitude wave motion of gas are given in [11] and [12]. Theoretical representation of the solution in the form of a series is also possible when a shock wave is formed, but in this case it will be required to retain a large number of series components for representing the discontinuous behavior of the function. The present investigation gives a technique for calculating the gas flow without imposing restrictions on the values of the Mach number and permits describing this flow with a discontinuity in the function behavior.

For numerical simulation of the gas flows in a tube, let us make use of the system of nonstationary narrow-channel equations written in the divergent form

$$\frac{\partial}{\partial t}(r\rho) + \frac{\partial}{\partial z}(r\rho u) + \frac{\partial}{\partial r}(r\rho v) = 0\,, \tag{1}$$

$$\frac{\partial}{\partial t}(r\rho u) + \frac{\partial}{\partial z}(r(\rho u^2 + p)) + \frac{\partial}{\partial r}(r\rho uv) - \frac{\partial}{\partial r}(r\tau) = 0\,, \tag{2}$$

$$\frac{\partial}{\partial t}(r\rho E)+\frac{\partial}{\partial z}(r\rho uH)+\frac{\partial}{\partial r}(r\rho vH)+\frac{\partial}{\partial r}(rq)-\frac{\partial}{\partial r}(ru\tau)=0\,. \tag{3}$$

The process of molecular transfer is described by the Newton law and the process of heat conduction — by the Fourier law

$$\tau=\mu\frac{\partial u}{\partial r}\,,\quad q=-\lambda\frac{\partial T}{\partial r}\,. \tag{4}$$

The relation between the gas-dynamic functions of the gas state in the elementary case is given in the form of the Mendeleev–Clapeyron law $p = \rho RT$ and the law for a calorically perfect gas C_v = const (pressure and temperature independence of the specific heat conductivity of the gas).

As boundary conditions for the gas velocity and temperature on the tube axis, we took the symmetry condition, and on the tube wall — the adhesion condition and the given temperature value. On the end surfaces, including the moving piston surface, the sealing and heat insulation conditions were given.

In analyzing cyclic systems, the initial conditions are immaterial, as a rule. In the given case, the solution was constructed from the state of a stationary gas with a temperature equal to the wall temperature. The piston was at the upper dead point (UDP).

To solve the system of equations (1)–(3), we employed an explicit–implicit scheme using a form of representation of the system of narrow-channel equations that is divergent with respect to the vector of the main variables and permits describing flows with discontinuities of the gas-dynamic functions. The derivatives in differentiating in the longitudinal direction were determined on the known lime layer, and in differentiating in the transverse direction they were determined implicitly. The region was discretized by the control volume approach. The flow values in the longitudinal direction through the control cell boundaries were determined by the Godunov method for solving the Riemann problem on the arbitrary discontinuity decay according to [13]. Approximation of the derivatives in the transverse direction was carried out by means of standard central-difference relations. This scheme sustains the first order of time approximation and an order not lower than the second one of space approximation. The algorithm of numerical integration of the system of equations (1)–(3) is described in more detail in [14].

In solving the problem, a grid uniform in z and thickened in r towards the channel wall is used. To provide thickening of the grid nodes towards the channel wall, we used the transformation of coordinates following the approach of [15]. The calculation was made on a fixed number of nodes. Therefore, the grid was rearranged in the longitudinal direction in accordance with the piston motion.

The calculation has been made for the following dimensions of the tube: L = 3 m; $D = 4.0{\cdot}10^{-3}$ m. The circular vibration frequency of the piston was $\omega = 330.5846$ sec^{-1} and corresponded to the first resonance harmonic of acoustic vibrations. The vibration amplitude of the piston was h = 0.3 m. The initial state of the gas p = 0.1 MPa, T = 300 K. The thermal state of the tube wall corresponded to the isothermal conditions of the gas flow with a temperature equal to T_w = 300 K. For numerical representation of the solution, we used a grid of size 301 × 31 nodes. The time integration step was chosen to be equal to $3.0{\cdot}10^{-6}$ sec.

The chosen geometric characteristics of the tube and the parameters of the gas flow exciter correspond to the laminar regime of the gas flow in it with the formation of the shock-wave structure of the flow. A small value of the Stokes number leads to a fast dynamic stabilization of the structure of a flow, from which the nonviscous flow core is practically absent, and a small cross section of the tube leads to a fast temperature stabilization. The latter is most important because only the termination of the formation of radial temperature gradients throughout the tube cross section provides complete energy balance in the system. This balance consists of the fact that in the cycle time transformation of the energy obtained from the moving piston to the energy of the gas compressed in the shock wave occurs with simultaneous heat removal by means of the heat transfer to the tube wall. In particular, after 20 cycles of piston oscillation the work done by it on the gas during a cycle differs from the quantity of heat removed through the tube wall by less than 0.1%. This points to the fact that the gas oscillations in the system have become stable.

Some of the integral characteristics of the cycle are presented in Fig. 1. The symbol U marks the curve of the current rate of motion of the piston, and N and Q correspond to the power at which the piston does work on the gas and the power of the length-total thermal flow through the tube wall. As one would expect, the heat transfer maxi-

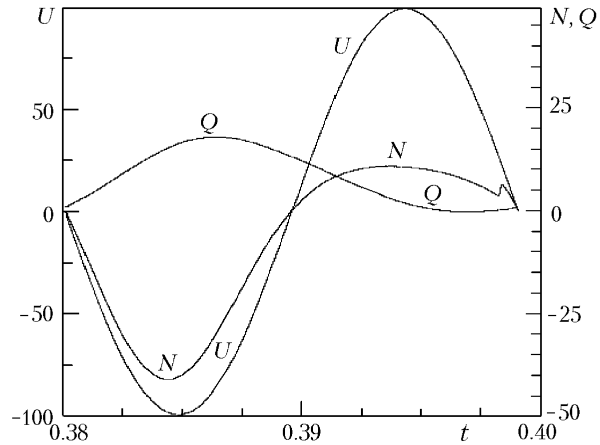

Fig. 1. Integral characteristics of the cycle. U, m/sec; N, Q, W; t, sec.

mum falls at the moment of the greatest compression of the gas in the tube with a small phase lag of oscillations. Oscillations of the volume-average gas pressure and the volume-average gas temperature lag behind the piston oscillations by an angle of $\pi/2$. It may be stated that after the start of the system from the UDP the average temperature during the first two cycles decreased, but subsequently its increase and stabilization at 304 K with an oscillation amplitude of 9 K occurred. In so doing, the average pressure in the tube increased by 10% with an oscillation amplitude of 11% of the average value. Such a displacement of the thermodynamic equilibrium point is due to the periodic change in the gas-column volume and the temperature conditions on the tube wall. The observed behavior of the power curve as the piston is doing work on the gas finds an explanation in analyzing the local characteristics of the system.

The current values of the local hydrodynamic variables are determined by their dependence on the oscillation frequency. Since gas oscillations occur at the near-resonance frequency, the values of the local characteristics differ widely from the average-volume ones. In particular, the average temperature in the vicinity of the stationary end of the tube is 315 K with an oscillation amplitude of 45 K. The average pressure at the tube end is 0.11 MPa, but because of the time asymmetry of the oscillation process the pressure maximum is 0.232 MPa and the minimum is 0.58 MPa. The pattern of the pressure oscillation near the piston surface is characterized by the fact that, with respect to the piston oscillations, the phase shift is about 180°, i.e., pressure oscillations occur out of phase. As a result of this, the power with which the piston does work on the gas has a considerable amplitude asymmetry about the time axis at different values of the cycle phase (1).

Let us illustrate the behavior of the gas-dynamic functions at characteristic points of the tube. Curves 1–5 in Fig. 1 correspond to equidistant points along the tube length, with the first monitoring point coinciding with the tube end and the fifth point coinciding with the piston surface. Figure 2 shows the time dependences of: (a) pressure (the current pressure is referred to the value at the initial instant of time), (b) velocity averaged over the tube cross section, (c) temperature averaged over the tube cross-section, and (d) specific thermal flux.

Analysis of the pressure–time diagram shows that the total amplitude of pressure oscillations in different cross sections of the tube differs but slightly, despite the considerably viscous character of the medium flow. At monitoring points 2–4, gas compression in the shock wave occurs twice as it propagates in opposite directions, and at points 1 and 5 single compression and reflection of the shock wave from the stationary end and the moving piston are observed. In the diagrams showing the dependence of the cross-section-average velocity on the longitudinal coordinate (Fig. 2b), the position of the shock wave is given as a sharp front of the change in the function, and its reflection from the end surfaces of the tube — as bursts disturbing the smooth behavior of the function. From the graph of variations in the cross-section-average temperature of the gas, it is seen that the local values of the function differ considerably from the tube wall temperature, and the largest oscillation amplitude falls at the end surfaces of the tube. Near the stationary end the absolute temperature maximum is registered, and near the piston surface, at the phase of rarefaction wave formation, the absolute minimum is observed. Variations in the gas temperature in the tube cross sections lead to interesting mechanisms of the thermal flow into the tube wall. As follows from Fig. 1, at the chosen parame-

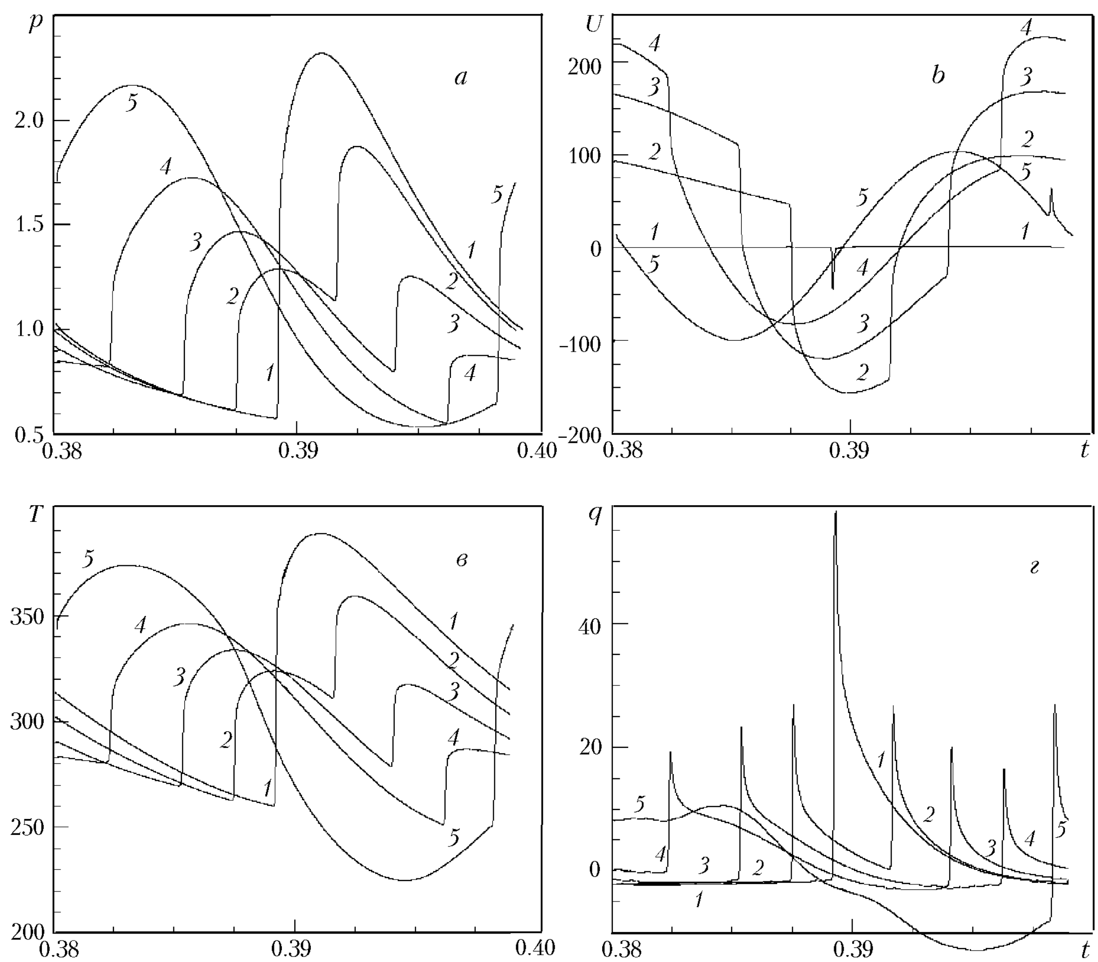

Fig. 2. Changes in the values of the gas-dynamic function at the monitoring points in a cycle. p, dimensionless; U, m/sec; T, K; q, kW/m^2; t, sec.

ters of the problem, the total thermal flux through the tube wall is positive at all times, i.e., from the gas the heat is transferred to the wall. In so doing, the dominant contribution in this transfer is made by the thermal flux immediately after the moving shock wave. However, the structure of the temperature boundary layer is such that at corresponding instants of time in the other parts of the tube before the shock wave the temperature of the tube wall exceeds the gas temperature in the vicinity of the wall; therefore, the thermal flux is negative. Under the conditions of a time-alternating thermal flux, where at certain instants of time the temperature profile in the tube cross section has a minimum, correct modeling of the thermal state for the material of the tube wall is necessary. In particular, for correct comparison of calculated and experimental results it is necessary to consider the boundary conditions on the tube wall as a solution of the problem of conjugate heat exchange. It may be noted that during the cycle time, in the region of the end, heat removal occurs, and in the region of the piston, vice versa, the gas is heated from the tube wall. Therefore, for a finite thickness of the tube wall axial thermal flows in the wall material should arise.

The results of the calculation reveal the following features of the shock-wave gas flow in a closed tube with a harmonic near-resonance law of piston oscillation. The thickness of the dynamic nonstationary boundary layer on the tube wall is determined by the piston oscillation frequency and the gas viscosity. Depending on the value of the Stokes number, the formation of a nonviscous flow core in the vicinity of the tube axis is possible. The nonstationary temperature boundary layer has a much more complex structure and covers the whole cross section of the tube. Therefore, the stabilization times of the structure of the dynamic and temperature boundary layers are different. Thus, for the investigated gas flow the known Reynolds analogy is not fulfilled.

For the given characteristics of the system, the formation of temperature stratification of the gas is observed (in the vicinity of the plugged end of the tube the gas is hotter compared to the gas in the vicinity of the piston). This leads to the formation of thermal flows in the axial direction.

NOTATION

a, thermal diffusivity of the gas; c, sound velocity; C_v, specific heat capacity at a constant volume; D, tube diameter; $E = C_v T + (u^2 + v^2)/2$, total specific energy; $H = E + p/\rho$, total specific enthalpy; h, oscillation amplitude of the piston; L, tube length; M, Mach number; N, power with which the piston does work on the gas; p, pressure; Pr, Prandtl number; Q, heat-transfer power between the gas and the tube wall; q, specific thermal flow; r, radial coordinate; R, gas constant; Re, Reynolds number; St, Stokes number; T, temperature; t, time; U, rate of motion of the piston; u and v, axial and radial components of the velocity vector; z, axial coordinate; λ, heat-conductivity coefficient of the gas; μ and ν, dynamic and kinematic viscosity coefficients; Θ, temperature factor; ρ, density; τ, friction stress; ω, oscillation frequency of the piston. Subscripts: 0 and w, values of the functions on the channel axis and wall, respectively.

REFERENCES

1. D. N. Popov, *Nonstationary Hydromechanical Processes* [in Russian], Mashinostroenie, Moscow (1982).
2. B. F. Glikman, *Mathematical Models of Pneumohydraulic Systems* [in Russian], Nauka, Moscow (1986).
3. E. P. Valueva, Pulsating turbulent gas flow in a circular tube, *Teplofiz. Vys. Temp.*, **40**, No. 1, 77–85 (2002).
4. A. L. Tukmakov, Generation of high-frequency harmonics under large-amplitude oscillations of a gas in a closed tube, *Inzh.-Fiz. Zh.*, **74**, No. 5, 39–43 (2001).
5. A. Hamed, K. Das, and D. Basu, Numerical simulation of unsteady flow in resonance tube, *AIAA J.*, No. 1118, 1–8 (2002).
6. S. G. Rubin, *Numerical and Physical Aspects of Aerodynamic Flows I*, Springer, New York–Berlin–Heidelberg (1981), pp. 171–186.
7. S. V. Patankar and D. B. Spalding, A calculation procedure for heat, mass and momentum transfer in three-dimensional parabolic flows, *Int. J. Heat Mass Transfer*, **15**, 1787–1806 (1972).
8. A. M. Bubenchikov and V. F. Kharlamov, *Mathematical Models of Flow and Heat Transfer in Internal Problems of the Viscous Gas Dynamics* [in Russian], Izd. Tomsk. Gos. Univ., Tomsk (1993).
9. S. V. Bulovich, V. E. Vikolainen, R. L. Petrov, and L. S. Polkovnikova, Numerical study of the formation of a shock-wave structure of the gas flow in a closed tube caused by harmonic oscillations of the piston, in: *Ext. Abstr. of XXth Jubilee Int. Seminar on Jet, Separated, and Nonstationary Flows* [in Russian], IPTs St. Petersburgsk. GUDT, St. Petersburg (2004), pp. 47–48.
10. S. V. Bulovich and A. L. Maslova, Mathematical modeling of resonance oscillations of gas in cylindrical closed tubes excited by harmonic oscillations of the piston, in: *Proc. Sci.-Practical Conf. and School-Seminar "Formation of Technical Policy of Innovation Highly Scientific Technologies"* [in Russian], Izd. Politekh. Univ., St. Petersburg (2004), pp. 226–234.
11. R. G. Galiullin, E. R. Galiullina, and E. I. Permyakov, Influence of absorption on nonlinear oscillations of gas in a closed tube, *Inzh.-Fiz. Zh.*, **68**, No. 3, 408–415 (1995).
12. R. G. Galiullin, R. G. Zaripov, E. R. Galiullina, and R. I. Davydov, Resonance oscillations of a gas in a closed tube in the region of transition to shock waves, *Inzh.-Fiz. Zh.*, **73**, No. 2, 370–375 (2000).
13. S. K. Godunov, A. V. Zabrodin, M. Ya. Ivanov, A. N. Kraiko, and G. P. Prokopov, *Numerical Solution of Multidimensional Problems of Gas Dynamics* [in Russian], Nauka, Moscow (1976).
14. S. V. Bulovich, Algorithms of numerical integration of the nonstationary system of narrow-channel, equations in: *Proc. Sci.-Practical Conf. and School-Seminar "Formation of Technical Policy of Innovation Highly Scientific Technologies"* [in Russian], Izd. Politekh. Univ., St. Petersburg (2004), pp. 188–207.
15. C. A. J. Fletcher, *Computational Techniques for Fluid Dynamics* [Russian translation], Vol. 2, Mir, Moscow (1991).